

ZBIRKA REŠENIH INTEGRALOV

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Basic Forms

$$(1) \quad \int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1$$

$$(2) \quad \int \frac{1}{x} dx = \ln x$$

$$(3) \quad \int u dv = uv - \int v du$$

$$(4) \quad \int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b|$$

Integrals of Rational Functions

$$(5) \quad \int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a}$$

$$(6) \quad \int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1} + c, \quad n \neq -1$$

$$(7) \quad \int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)}$$

$$(8) \quad \int \frac{1}{1+x^2} dx = \tan^{-1} x$$

$$(9) \quad \int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$(10) \quad \int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2|$$

$$(11) \quad \int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a}$$

$$(12) \quad \int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2|$$

$$(13) \quad \int \frac{1}{ax^2 + bx + c} dx = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}}$$

$$(14) \quad \int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \quad a \neq b$$

$$(15) \quad \int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x|$$

$$(16) \quad \int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{a\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}}$$

Integrals with Roots

$$(17) \quad \int \sqrt{x-a} dx = \frac{2}{3}(x-a)^{3/2}$$

$$(18) \quad \int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a}$$

$$(19) \quad \int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x}$$

$$(20) \quad \int x\sqrt{x-a} dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, & \text{or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, & \text{or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases}$$

$$(21) \quad \int \sqrt{ax+b} dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{ax+b}$$

$$(22) \quad \int (ax+b)^{3/2} dx = \frac{2}{5a}(ax+b)^{5/2}$$

$$(23) \quad \int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a}$$

$$(24) \quad \int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a}$$

$$(25) \quad \int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a \ln [\sqrt{x} + \sqrt{x+a}]$$

$$(26) \quad \int x\sqrt{ax+b} dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b}$$

$$(27) \quad \int \sqrt{x(ax+b)} dx = \frac{1}{4a^{3/2}} \left[(2ax+b)\sqrt{ax(ax+b)} - b^2 \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| \right]$$

$$(28) \quad \int \sqrt{x^3(ax+b)} dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right|$$

$$(29) \quad \int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right|$$

$$(30) \quad \int \sqrt{a^2 - x^2} dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$$

$$(31) \quad \int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$$

$$(32) \quad \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right|$$

$$(33) \quad \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}$$

$$(34) \quad \int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2}$$

$$(35) \quad \int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$$

$$(36) \quad \int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right|$$

$$(37) \quad \int \sqrt{ax^2 + bx + c} dx = \frac{b + 2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right|$$

$$(38) \quad \int x\sqrt{ax^2 + bx + c} dx = \frac{1}{48a^{5/2}} \left(2\sqrt{a}\sqrt{ax^2 + bx + c} (-3b^2 + 2abx + 8a(c + ax^2)) \right. \\ \left. + 3(b^3 - 4abc) \ln \left| b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + c} \right| \right)$$

$$(39) \quad \int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right|$$

$$(40) \quad \int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{a}\sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right|$$

$$(41) \quad \int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}}$$

Integrals with Logarithms

$$(42) \quad \int \ln ax dx = x \ln ax - x$$

$$(43) \quad \int x \ln x dx = \frac{1}{2}x^2 \ln x - \frac{x^2}{4}$$

$$(44) \quad \int x^2 \ln x dx = \frac{1}{3}x^3 \ln x - \frac{x^3}{9}$$

$$(45) \quad \int x^n \ln x dx = x^{n+1} \left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2} \right), \quad n \neq -1$$

$$(46) \quad \int \frac{\ln ax}{x} dx = \frac{1}{2} (\ln ax)^2$$

$$(47) \quad \int \frac{\ln x}{x^2} dx = -\frac{1}{x} - \frac{\ln x}{x}$$

$$(48) \quad \int \ln(ax + b) dx = \left(x + \frac{b}{a} \right) \ln(ax + b) - x, \quad a \neq 0$$

$$(49) \quad \int \ln(x^2 + a^2) dx = x \ln(x^2 + a^2) + 2a \tan^{-1} \frac{x}{a} - 2x$$

$$(62) \quad \int x^2 e^x dx = (x^2 - 2x + 2) e^x$$

$$(63) \quad \int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax}$$

$$(64) \quad \int x^3 e^x dx = (x^3 - 3x^2 + 6x - 6) e^x$$

$$(65) \quad \int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$(66) \quad \int x^n e^{ax} dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1 + n, -ax], \quad \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$$

where

$$(67) \quad \int e^{ax^2} dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(ix\sqrt{a})$$

$$(68) \quad \int e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(x\sqrt{a})$$

$$(69) \quad \int x e^{-ax^2} dx = -\frac{1}{2a} e^{-ax^2}$$

$$(70) \quad \int x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2}$$

Integrals with Trigonometric Functions

$$(71) \quad \int \sin ax dx = -\frac{1}{a} \cos ax$$

$$(72) \quad \int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$(73) \quad \int \sin^3 ax dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a}$$

$$(74) \quad \int \sin^n ax \, dx = -\frac{1}{a} \cos ax \, {}_2F_1 \left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax \right]$$

$$(75) \quad \int \cos ax \, dx = \frac{1}{a} \sin ax$$

$$(76) \quad \int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$(77) \quad \int \cos^3 ax \, dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a}$$

$$(78) \quad \int \cos^p ax \, dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1 \left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax \right]$$

$$(79) \quad \int \cos x \sin x \, dx = \frac{1}{2} \sin^2 x + c_1 = -\frac{1}{2} \cos^2 x + c_2 = -\frac{1}{4} \cos 2x + c_3$$

$$(80) \quad \int \cos ax \sin bx \, dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b$$

$$(81) \quad \int \sin^2 ax \cos bx \, dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$$

$$(82) \quad \int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x$$

$$(83) \quad \int \cos^2 ax \sin bx \, dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$$

$$(84) \quad \int \cos^2 ax \sin ax \, dx = -\frac{1}{3a} \cos^3 ax$$

$$(85) \quad \int \sin^2 ax \cos^2 bx \, dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$$

$$(86) \quad \int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

$$(87) \quad \int \tan ax \, dx = -\frac{1}{a} \ln \cos ax$$

$$(88) \quad \int \tan^2 ax \, dx = -x + \frac{1}{a} \tan ax$$

$$(89) \quad \int \tan^n ax \, dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1 \left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax \right)$$

$$(90) \quad \int \tan^3 ax \, dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax$$

$$(91) \quad \int \sec x \, dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2} \right)$$

$$(92) \quad \int \sec^2 ax \, dx = \frac{1}{a} \tan ax$$

$$(93) \quad \int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x|$$

$$(94) \quad \int \sec x \tan x \, dx = \sec x$$

$$(95) \quad \int \sec^2 x \tan x \, dx = \frac{1}{2} \sec^2 x$$

$$(96) \quad \int \sec^n x \tan x \, dx = \frac{1}{n} \sec^n x, n \neq 0$$

$$(97) \quad \int \csc x \, dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C$$

$$(98) \quad \int \csc^2 ax \, dx = -\frac{1}{a} \cot ax$$

$$(99) \quad \int \csc^3 x \, dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x|$$

$$(100) \quad \int \csc^n x \cot x \, dx = -\frac{1}{n} \csc^n x, n \neq 0$$

$$(101) \quad \int \sec x \csc x \, dx = \ln |\tan x|$$

Products of Trigonometric Functions and Monomials

$$(102) \quad \int x \cos x \, dx = \cos x + x \sin x$$

$$(103) \quad \int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$$

$$(104) \quad \int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2) \sin x$$

$$(105) \quad \int x^2 \cos ax \, dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax$$

$$(106) \quad \int x^n \cos x \, dx = -\frac{1}{2} (i)^{n+1} [\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)]$$

$$(107) \quad \int x^n \cos ax \, dx = \frac{1}{2} (ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, iax)]$$

$$(108) \quad \int x \sin x \, dx = -x \cos x + \sin x$$

$$(109) \quad \int x \sin ax \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2}$$

$$(110) \quad \int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x$$

$$(111) \quad \int x^2 \sin ax \, dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2}$$

$$(112) \quad \int x^n \sin x \, dx = -\frac{1}{2} (i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, ix)]$$

$$(113) \quad \int x \cos^2 x \, dx = \frac{x^2}{4} + \frac{1}{8} \cos 2x + \frac{1}{4} x \sin 2x$$

$$(114) \quad \int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{1}{8} \cos 2x - \frac{1}{4} x \sin 2x$$

$$(115) \quad \int x \tan^2 x \, dx = -\frac{x^2}{2} + \ln \cos x + x \tan x$$

$$(116) \quad \int x \sec^2 x \, dx = \ln \cos x + x \tan x$$

Products of Trigonometric Functions and Exponentials

$$(117) \quad \int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x)$$

$$(118) \quad \int e^{bx} \sin ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax)$$

$$(119) \quad \int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x)$$

$$(120) \quad \int e^{bx} \cos ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax)$$

$$(121) \quad \int x e^x \sin x \, dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x)$$

$$(122) \quad \int x e^x \cos x \, dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x)$$

Integrals of Hyperbolic Functions

$$(123) \quad \int \cosh ax \, dx = \frac{1}{a} \sinh ax$$

$$(124) \quad \int e^{ax} \cosh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases}$$

$$(125) \quad \int \sinh ax \, dx = \frac{1}{a} \cosh ax$$

$$(126) \quad \int e^{ax} \sinh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases}$$

$$(127) \quad \int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax$$

$$(128) \quad \int e^{ax} \tanh bx \, dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)^2} {}_2F_1 \left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx} \right] \\ \quad - \frac{1}{a} e^{ax} {}_2F_1 \left[\frac{a}{2b}, 1, 1E, -e^{2bx} \right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases}$$

$$(129) \quad \int \cos ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx]$$

$$(130) \quad \int \cos ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx]$$

$$(131) \quad \int \sin ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx]$$

$$(132) \quad \int \sin ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx]$$

$$(133) \quad \int \sinh ax \cosh ax \, dx = \frac{1}{4a} [-2ax + \sinh 2ax]$$

$$(134) \quad \int \sinh ax \cosh bx \, dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx]$$