**ODVOD**

Po definiciji izračunaj $f'(x)$ :

1. $f(x) = \frac{1}{x}$, R: $-\frac{1}{x^2}$
2. $f(x) = \sin x$, R: $\cos x$
3. $f(x) = 3x^2$, R: $6x$
4. $f(x) = 2x - 3$, R: 2

Poišči odvode naslednjim funkcijam:

1. $f(x) = 2x^3 - 3x^2 + 4$, R: $6x^2 - 6x$
2. $f(x) = \frac{x^3}{3} - \frac{x^4}{4}$, R: $x^2 - x^3$
3. $f(x) = \frac{x-1}{x+1}$, R: $\frac{2}{(x+1)^2}$
4. $f(x) = \frac{x^2+x+1}{x^2-x+1}$, R: $\frac{-2x^2+2}{(x^2-x+1)^2}$
5. $f(x) = 2\sqrt{x}$, R: $\frac{1}{\sqrt{x}}$
6. $f(x) = x\sqrt{x}$, R: $\frac{3}{2}x^{\frac{1}{2}}$
7. $f(x) = e^x(x^2 - 2x + 2)$, R: $x^2 e^x$
8. $f(x) = x \sin x$, R: $\sin x + x \cos x$
9. $f(x) = x^2 \sin x + 2x \cos x - 2 \sin x$, R: $x^2 \cos x$
10. $f(x) = x \ln x - x$, R: $\ln x$
11. $f(x) = \frac{x^3}{3} \ln x - \frac{x^3}{9}$, R: $x^2 \ln x$
12. $f(x) = 2 \sin 6x$, R: $12 \cos 6x$
13. $f(x) = (3x + 8)^n$, R: $3n(3x + 8)^{n-1}$
14. $f(x) = e^{-3x}$, R: $-3e^{-3x}$
15. $f(x) = (a^2 - x^2)^{\frac{1}{2}}$, $a = konst$, R: $-x(a^2 - x^2)^{-\frac{1}{2}}$
16. $f(x) = \ln(x^3 + x)$, R: $\frac{3x^2+1}{x^3+x}$
17. $f(x) = \cos ax \sin bx$, $a$ in $b$ sta konstanti, R: $-a \sin ax \sin bx + b \cos ax \cos bx$
18. $f(x) = \ln tgx$, R: $\frac{1}{\sin x \cos x}$
19. $f(x) = \frac{x^4}{4}(\ln^2 x - \ln \sqrt{x} + \frac{1}{8})$; R: $x^3 \ln^2 x$
20. $f(x) = e^{2\sqrt{ax}}$, $a = konst$, R: $a(ax)^{-\frac{1}{2}} e^{2\sqrt{ax}}$
21. $f(x) = \arcsin ax$, R: $\frac{a}{\sqrt{1-a^2x^2}}$
22. $f(x) = (\arcsin x)^2$, R: $2 \arcsin \frac{1}{\sqrt{1-x^2}}$
23. $f(x) = \arccos(a - x)$, R: $\frac{1}{\sqrt{1-(a-x)^2}}$
24. $f(x) = arctg\frac{x+1}{x-1}$, R: $\frac{-1}{x^2+1}$
25. $f(x) = 6^{3x}$, R: $6^{3x} \ln 6 * 3$

26. $f(x) = \ln(\cos(x^4 + 4x))$, R: $-(4x^3 + 4) * tg(x^4 + 4x)$

27. $f(x) = \arctan(n * tgx)$, R: $\frac{n}{\cos^2 x + n^2 \sin^2 x}$

Vaje

(1) $(2x^3-3x^2+4)' = \boxed{6x^2-6x}$

(2) $\left(\frac{x^3}{3}-\frac{x^4}{4}\right)' = \frac{1}{3}\cdot 3x^2 - \frac{1}{4}\cdot 4x^3 = \boxed{x^2-x^3}$

(3) $\left(\frac{x-1}{x+1}\right)' = \frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2} = \frac{x+1-x+1}{(x+1)^2} = \boxed{\frac{2}{(x+1)^2}}$

(4) $\left(\frac{x^2+x+1}{x^2-x+1}\right)' = \frac{(2x+1)(x^2-x+1)-(x^2+x+1)(2x-1)}{(x^2-x+1)^2} = \frac{2x^3-2x^2+2x+x^2-x+1-(2x^3-x^2+2x^2-x+2x-1)}{(x^2-x+1)^2}$

$\boxed{=\frac{-2x^2+2}{(x^2-x+1)^2}}$

(5) $(2\sqrt{x})' = (2\cdot x^{\frac{1}{2}})' = 2\,\frac{1}{2}\,x^{-\frac{1}{2}} = x^{-\frac{1}{2}} = \boxed{\frac{1}{\sqrt{x}}}$

(6) $(x\sqrt{x})' = (x\cdot x^{\frac{1}{2}})' = (x^{\frac{3}{2}})' = \boxed{\frac{3}{2}x^{\frac{1}{2}}}$

(7) $(e^x(x^2-2x+2))' = e^x\cdot(x^2-2x+2) + e^x(2x-2) = e^x x^2 - 2e^x x + 2e^x + 2e^x x - 2e^x = \boxed{e^x x^2}$

(8) $(x\sin x)' = \boxed{\sin x + x\cos x}$

(9) $(x^2\sin x + 2x\cos x - 2\sin x)' = 2x\sin x + x^2\cos x + 2\cos x - 2x\sin x - 2\cos x = \boxed{x^2\cos x}$

(10) $(x\cdot\ln x - x)' = \ln x + x\frac{1}{x} - 1 = \boxed{\ln x}$

(11) $\left(\frac{x^3}{3}\cdot\ln x - \frac{x^3}{9}\right)' = \left(\frac{x^3\ln x}{3} - \frac{x^3}{9}\right)' = \frac{1}{3}3x^2\ln x + \frac{1}{3}x^3\frac{1}{x} - \frac{1}{9}3x^2 = x^2\ln x + \frac{x^2}{3} - \frac{x^2}{3}$

$= x^2\ln x$

(12) $(2\sin 6x)' = 2\cos 6x\cdot 6 = \boxed{12\cos 6x}$

(13) $((3x+8)^n)' = n(3x+8)^{n-1}\cdot 3 = \boxed{3n(3x+8)^{n-1}}$

(14) $(e^{-3x})' = e^{-3x}\cdot(-3) = \boxed{-3e^{-3x}}$

15. $a = \text{konst.}$ $\left(\left(a^2 - x^2\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\left(a^2 - x^2\right)^{-\frac{1}{2}} \cdot (-2x) = \boxed{-x\left(a^2 - x^2\right)^{-\frac{1}{2}}}$

16. $\left(\ln\left(x^3 + x\right)\right)' = \frac{1}{x^3 + x} \cdot \left(3x^2 + 1\right) = \boxed{\frac{3x^2 + 1}{x^3 + x}}$

17. $(\cos ax \sin bx)' = \boxed{-\sin ax \cdot a \cdot \sin bx + \cos ax \cdot \cos bx \cdot b}$

$a, b - \text{konst.}$

18. $(\ln \operatorname{tg} x)' = \frac{1}{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x} = \frac{1}{\frac{\sin x}{\cos x}} \cdot \frac{1}{\cos^2 x} = \boxed{\frac{1}{\sin x \cos x}}$

19. $\frac{x^4}{4}\left(\ln^2 x - \ln\sqrt{x} + \frac{1}{8}\right) = ?$ → rješena na koncu

20. $\left(e^{2\sqrt{ax}}\right)' = e^{2\sqrt{ax}} \cdot (ax)^{-\frac{1}{2}} \cdot a \cdot 1 = \boxed{a(ax)^{-\frac{1}{2}} \cdot e^{2\sqrt{ax}}}$

$a = \text{konst.}$

21. $(\arcsin ax)' = \frac{1}{\sqrt{1 - (ax)^2}} \cdot a = \boxed{\frac{a}{\sqrt{1 - a^2x^2}}}$

22. $\left((\arcsin x)^2\right)' = 2\arcsin x \cdot \frac{1}{\sqrt{1 - x^2}} = \boxed{\frac{2\arcsin x}{\sqrt{1 - x^2}}}$

23. $\left(\arccos(a - x)\right)' = \frac{-1}{\sqrt{1 - (a - x)^2}} \cdot (-1) = \boxed{\frac{1}{\sqrt{1 - (a - x)^2}}}$

24. $\left(\operatorname{arctg}\frac{x+1}{x-1}\right)' = \frac{1}{1 + \left(\frac{x+1}{x-1}\right)^2} \cdot \frac{x - 1 - (x + 1)}{(x - 1)^2} = \frac{-2}{(x-1)^2 + (x+1)^2} =$

$= \frac{-2}{x^2 - 2x + 1 + x^2 + 2x + 1} = \frac{-2}{2\left(x^2 + 1\right)} = \boxed{-\frac{1}{x^2 + 1}}$

25. $\left(6^{3x}\right)' = \boxed{6^{3x} \ln 6 \cdot 3}$

26. $\left(\ln\left(\cos\left(x^4 + 4x\right)\right)\right)' = \frac{1}{\cos\left(x^4 + 4x\right)} \cdot \left(-\sin\left(x^4 + 4x\right)\right) \cdot \left(4x^3 + 4\right) = \frac{\sin\left(x^4 + 4x\right)}{\cos\left(x^4 + 4x\right)} \cdot \left(4x^3 + 4\right) = \operatorname{tg}\left(x^4 + 4x\right) \cdot \left(4x^3 + 4\right)$

(27) $$\left(\operatorname{arctg}(n\cdot \operatorname{tg}x)\right)' = \frac{1}{1+(n\cdot \operatorname{tg}x)^2}\cdot n\,\frac{1}{\cos^2x} = \frac{n}{\left(1+\left(\frac{n\sin x}{\cos x}\right)^2\right)\cdot\cos^2x}$$

$$= \frac{n}{\cos^2x+\frac{n^2\sin^2x\,\cos^2x}{\cos^2x}} = \boxed{\frac{n}{\cos^2x+n^2\sin^2x}}$$

(19) $$\left(\frac{x^4}{4}\left(\ln^2x-\ln\sqrt{x}+\frac{1}{8}\right)\right)' =$$

$$\ln^2x = (\ln x)^2 = \left(2\ln x\cdot\frac{1}{x}\right) = \frac{2\ln x}{x}$$

$$\ln x^{\frac{1}{2}} = \left(\frac{1}{2}\ln x\right)' = \frac{1}{2}\,\frac{1}{x} = \frac{1}{2x}$$

$$= \frac{1}{4}4x^3\left(\ln^2x-\ln x^{\frac{1}{2}}+\frac{1}{8}\right)+\frac{x^4}{4}\cdot\left(\frac{2\ln x}{x}-\frac{1}{2x}\right)$$

$$= x^3\ln^2x - x^3\ln x^{\frac{1}{2}}+x^3\cdot\frac{1}{8}+\frac{x^4}{4}\cdot\frac{2\ln x}{x}-\frac{x^4}{4}\cdot\frac{1}{2x}$$

$$= x^3\ln^2x-\frac{x^3\ln x}{2}+\frac{x^3}{8}+\frac{2x^4\ln x}{4x}-\frac{x^4}{8x}$$

$$= \boxed{x^3\ln^2x}$$