1. Zapiši kompozituma funkcij: $f \circ g$ in $g \circ f$, če sta f in g realni funkciji s predpisoma:

a) $f_{(x)} = \cos x + 1 \text{in } g_{(x)} = x^2 + 5x + 2$

b) $f_{(x)} = e^x \text{ in } g_{(x)} = \dfrac{-1}{x^2}$

a) 1.

$$(f \circ g)(x) = f\left(g_{(x)}\right) = f\left(x^2 + 5x + 2\right) = \cos\left(x^2 + 5x + 2\right) + 1$$

2.

$$(g \circ f)(x) = f\left(g_{(x)}\right) = g\left(\cos x + 1\right) = \left(\cos x + 1\right)^2 + 5\left(\cos x + 1\right) + 2$$

$$= \cos^2 x + 2\cos x + 1 + 5\cos x + 5 + 2$$

$$= \cos^2 x + 7\cos x + 8$$

b) 1.

$$(f \circ g)(x) = f\left(g_{(x)}\right) = f\left(\frac{-1}{x^2}\right) = e^{\frac{-1}{x^2}} = \frac{1}{e^{\frac{1}{x^2}}}$$

2.

$$(g \circ f)(x) = f\left(g_{(x)}\right) = g\left(\frac{-1}{x^2}\right) = \frac{-1}{\left(e^x\right)^2} = -\frac{1}{e^{2x}}$$

2. Zapiši kompozituma funkcij: $f \circ g \circ h$ in $h \circ g \circ f$, za $f_{(x)} = e^x$, $g_{(x)} = \dfrac{-1}{x^2}$ ter $h_{(x)} = \ln x^3$.

a) $$(f \circ g \circ h)(x) = f\left(g\left(h_{(x)}\right)\right) = f\left(g\left(\ln x^3\right)\right) = f\left(\frac{-1}{\left(\ln x^3\right)^2}\right) = e^{\frac{-1}{\left(\ln x^3\right)^2}} = e^{-\frac{1}{\ln x^6}} = \frac{1}{e^{\frac{1}{\ln x^6}}}$$

b) $$(h \circ g \circ f)(x) = h\left(g\left(f_{(x)}\right)\right) = h\left(g\left(e^x\right)\right) = h\left(\frac{-1}{\left(e^x\right)^2}\right) = \ln\left(\frac{-1}{e^{2x}}\right)^3 = \ln\left(\frac{-1}{e^{6x}}\right)$$

3. Naj bo $f : \mathbb{R} \to \mathbb{R}$, $f_{(x)} = x^2$. Zapiši:

a) $f \circ f$

b) $f \circ f \circ f$

a) $$(f \circ f)(x) = f\left(f_{(x)}\right) = f\left(x^2\right) = \left(x^2\right)^2 = x^4$$

b) $$(f \circ f \circ f)(x) = f\left(f\left(f_{(x)}\right)\right) = f\left(f\left(x^2\right)\right) = f\left(\left(x^2\right)^2\right) = \left(\left(x^2\right)^2\right)^2 = x^8$$

4. K dani funkciji poišči inverzno: $y=\frac{4x-5}{x-3}$.

$$y=\frac{4x-5}{x-3}$$

N:

$$4x-5=0$$
$$4x=5$$
$$x=\frac{5}{4}$$

P:

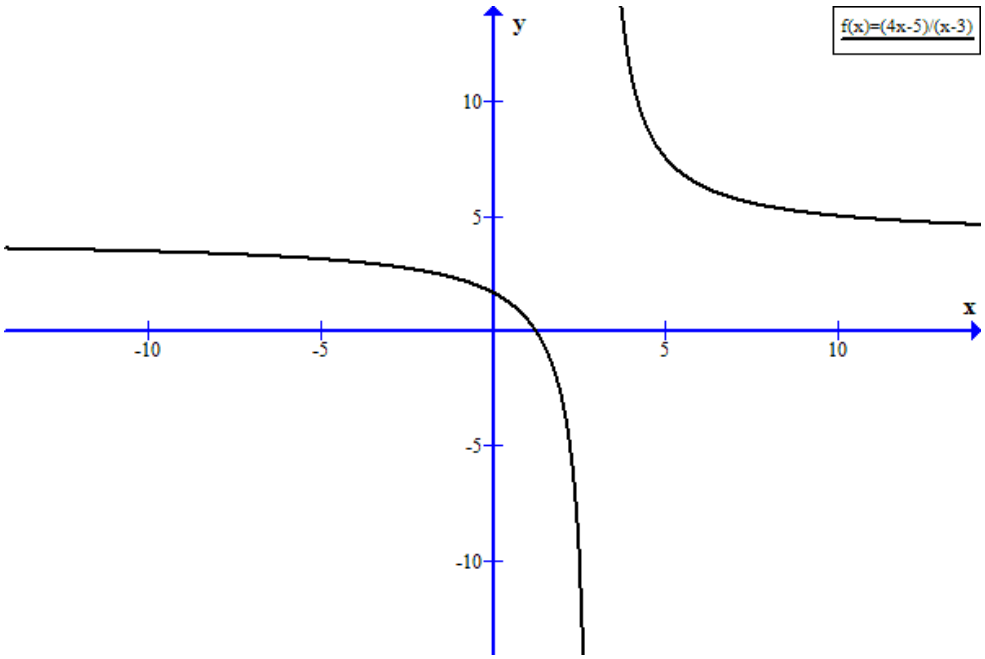
$$x-3=0$$
$$x=3$$

$$f_{(0)}=\frac{5}{3}$$

Asimptota:

$$y=\frac{4}{1}=4$$

$$Df:\mathbb{R}\setminus\{3\}$$



Inverz funkcije obstaja le takrat, kadar je funkcija bijektivna (surjektivna in injektivna hkrati). Funkcija bo bijektivna tedaj, ko:

$$f:\mathbb{R}\setminus\{3\}\to\mathbb{R}\setminus\{4\}$$

$$y=\frac{4x-5}{x-3}$$
$$x=\frac{4y-5}{y-3}$$
$$xy-3x=4y-5$$
$$xy-4y=3x-5$$
$$y(x-4)=3x-5$$
$$y'=\frac{3x-5}{x-4}$$

5. Nariši grafa funkcij:

a) $y=\dfrac{(2x-3)^2}{x^2-3x+2}$

b) $f_{(x)}=\begin{cases} x^2, & |x|<2 \\ 2x+1, & |x|>2 \\ 2, & |x|=2 \end{cases}$

a) .

$$y=\frac{(2x-3)^2}{x^2-3x+2}=\frac{4x^2-12x+9}{x^2-3x+2}$$

N:

$$4x^2-12x+9=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a\ \sqrt{}}$$

$$x_{1,2}=\frac{-(-12)\pm\sqrt{(-12)^2-4\cdot4\cdot9}}{2\cdot4\sqrt{}}$$

$$x_{1,2}=\frac{3}{2}$$

P:

$$x^2-3x+2=0$$

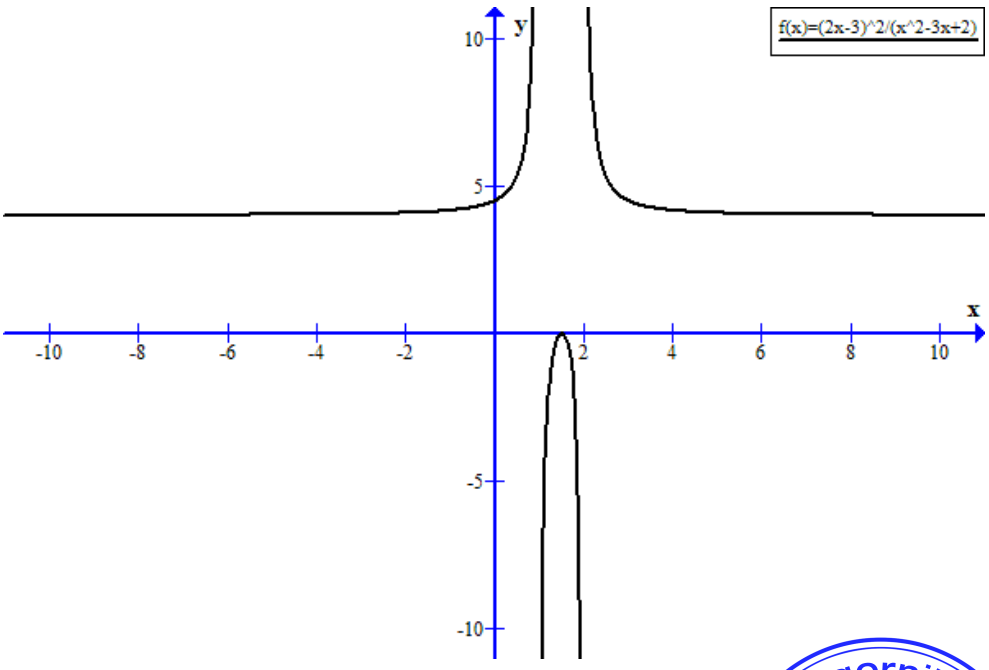
$$x_1=2$$

$$x_2=1$$

$$f_{(0)}=\frac{9}{2}$$

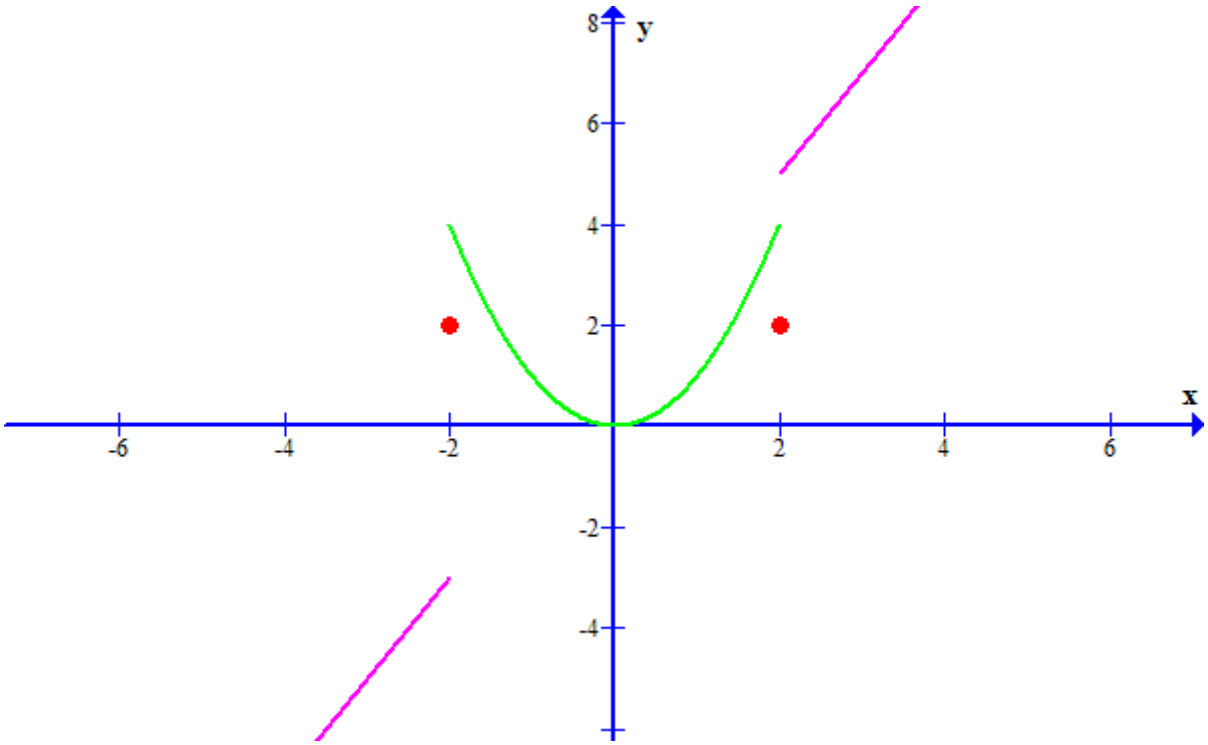
Asimptota:

$$y=\frac{4}{1}=4$$



b) .

$$f_{(x)} = \begin{cases} x^2, & |x| < 2 \\ 2x+1, & |x| > 2 \\ 2, & |x| = 2 \end{cases}$$



6. Izračunaj limite:

a) $\lim_{x\to 0}\left(\frac{\ln\cos x}{x}+\frac{e^x-1}{\sin x}\right)$

b) $\lim_{x\to 1}\frac{x^2+x-2}{2x^2-x-1}$

c) $\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+4x-1}-\sqrt{x^3+2x+1}}$

a) $\lim_{x\to 0}\left(\frac{\ln\cos x}{x}+\frac{e^x-1}{\sin x}\right)=\lim_{x\to 0}\frac{\ln\cos x}{x}+\lim_{x\to 0}\frac{e^x-1}{\sin x}\overset{\text{L'H}}{=}\lim_{x\to 0}\frac{\frac{1}{\cos x}\cdot(-\sin x)}{1}+\lim_{x\to 0}\frac{e^x}{\cos x}$

$\lim_{x\to 0}\frac{\frac{1}{\cos 0}\cdot(-\sin 0)}{1}+\lim_{x\to 0}\frac{e^0}{\cos 0}=\lim_{x\to 0}\frac{\frac{1}{1}\cdot 0}{1}+\lim_{x\to 0}\frac{1}{1}=\frac{1}{1}=1$

b) $\lim_{x\to 1}\frac{x^2+x-2}{2x^2-x-1}\overset{\text{L'H}}{=}\lim_{x\to 1}\frac{2x+1}{2\cdot 2x-1}=\lim_{x\to 1}\frac{2x+1}{4x-1}=\lim_{x\to 1}\frac{2\cdot 1+1}{4\cdot 1-1}=\frac{3}{3}=1$

c)

$$\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+4x-1}-\sqrt{x^3+2x+1}}=\lim_{x\to 1}\frac{x-1}{\left(x^2+4x-1\right)^{\frac{1}{2}}-\left(x^3+2x+1\right)^{\frac{1}{2}}}$$

$$\overset{\text{L'H}}{=}\lim_{x\to 1}\frac{1}{\frac{1}{2}\cdot\left(x^2+4x-1\right)^{-\frac{1}{2}}\cdot(2x+4)-\frac{1}{2}\cdot\left(x^3+2x+1\right)^{-\frac{1}{2}}\cdot 3x^2+2}$$

$$=\lim_{x\to 1}\frac{1}{\frac{1}{2}\cdot\left(1^2+4\cdot 1-1\right)^{-\frac{1}{2}}\cdot(2\cdot 1+4)-\frac{1}{2}\cdot\left(1^3+2\cdot 1+1\right)^{-\frac{1}{2}}\cdot 3\cdot 1^2+2}$$

$$=\lim_{x\to 1}\frac{1}{\frac{1}{2}\cdot\frac{1}{2}\cdot 6-\frac{1}{2}\cdot\frac{1}{2}\cdot 5}=\lim_{x\to 1}\frac{1}{\frac{6}{4}-\frac{5}{4}}=\frac{1}{\frac{1}{4}}=4$$