1 Dokaz zvez hiperboličnih funkcij:

1. Dokaz zveze $\mathrm{ch}^2 - \mathrm{sh}^2 = 1$:

$$\mathrm{ch}^2 - \mathrm{sh}^2 = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} - \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4}$$
$$= \frac{e^{2x} + 2e^x e^{-x} + e^{-2x} - e^{2x} + 2e^x e^{-x} - e^{-2x}}{4} = \frac{4e^x e^{-x}}{4} = e^x e^{-x} = e^{x-x} = e^0 = 1$$

2. Dokaz zveze $\mathrm{sh}(\mathrm{x+y}) = \mathrm{sh}(\mathrm{x}) \cdot \mathrm{ch}(\mathrm{y}) + \mathrm{sh}(\mathrm{y}) \cdot \mathrm{ch}(\mathrm{x})$:

$$\mathrm{sh}(x) \cdot \mathrm{ch}(y) + \mathrm{sh}(y) \cdot \mathrm{ch}(x) = \left(\frac{e^x - e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^y - e^{-y}}{2} \cdot \frac{e^x + e^{-x}}{2}\right) =$$
$$\frac{e^{x+y} + e^{x-y} - e^{y-x} - e^{-x-y} + e^{x+y} + e^{y-x} - e^{x-y} - e^{-y-x}}{4} = \frac{2e^{x+y} - 2e^{-x-y}}{4} =$$
$$\frac{e^{x+y} - e^{-x-y}}{2} = \frac{1}{2}\left(e^{x+y} - e^{-x-y}\right)$$

$$\mathrm{sh}(x) = \frac{e^x - e^{-x}}{2}$$
$$\mathrm{sh}(x+y) = \frac{e^{x+y} - e^{-(x+y)}}{2} = \frac{e^{x+y} - e^{-x-y}}{2} = \frac{1}{2}\left(e^{x+y} - e^{-x-y}\right)$$

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3. Dokaz zveze $\mathrm{ch}(\mathrm{x+y}) = \mathrm{ch}(\mathrm{x}) \cdot \mathrm{ch}(\mathrm{y}) - \mathrm{sh}(\mathrm{y}) \cdot \mathrm{sh}(\mathrm{x})$:

$$\mathrm{ch}(x) \cdot \mathrm{ch}(y) - \mathrm{sh}(y) \cdot \mathrm{sh}(x) = \left(\frac{e^x + e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2}\right) - \left(\frac{e^x - e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2}\right) =$$
$$\frac{e^{x+y} + e^{x-y} + e^{y-x} + e^{-x-y} - \left(e^{x+y} - e^{x-y} - e^{y-x} + e^{-x-y}\right)}{4} =$$
$$\frac{e^{x+y} + e^{x-y} + e^{y-x} + e^{-x-y} - e^{x+y} + e^{x-y} + e^{y-x} - e^{-x-y}}{4} =$$
$$\frac{e^{x-y} + e^{y-x} + e^{x-y} + e^{y-x}}{4} = \frac{2e^{x-y} + 2e^{y-x}}{4} = \frac{e^{x-y} + e^{y-x}}{2} =$$
$$\frac{1}{2}\left(e^{x-y} + e^{y-x}\right);\ \mathrm{ch}(x+y) = \mathrm{ch}(x) \cdot \mathrm{ch}(y) - \mathrm{sh}(y) \cdot \mathrm{sh}(x) \sim \text{ Zveza ne drži!}$$

Saj je;

$$\mathrm{ch}(x) = \frac{e^x + e^{-x}}{2}$$
$$\mathrm{ch}(x+y) = \frac{e^{x+y} + e^{-(x+y)}}{2} = \frac{e^{x+y} + e^{-x-y}}{2} = \frac{1}{2}\left(e^{x+y} + e^{-x-y}\right)$$

Dokaz, da drži zveza $\mathrm{ch}(x+y)=\mathrm{ch}(x)\cdot\mathrm{ch}(y)+\mathrm{sh}(y)\cdot\mathrm{sh}(x)$:

$$\mathrm{ch}(x)\cdot\mathrm{ch}(y)-\mathrm{sh}(y)\cdot\mathrm{sh}(x)=\left(\frac{e^{x}+e^{-x}}{2}\cdot\frac{e^{y}+e^{-y}}{2}\right)+\left(\frac{e^{x}-e^{-x}}{2}\cdot\frac{e^{y}-e^{-y}}{2}\right)=$$

$$\frac{e^{x+y}+e^{x-y}+e^{y-x}+e^{-x-y}+e^{x+y}-e^{x-y}-e^{y-x}+e^{-x-y}}{4}=$$

$$\frac{e^{x+y}+e^{y-x}+e^{-x+y}+e^{-y-x}}{4}=\frac{2e^{x+y}+2e^{-y-x}}{4}=$$

$$\frac{1}{2}\left(e^{x+y}+e^{-y-x}\right)$$

$$\mathrm{ch}(x)=\frac{e^{x}+e^{-x}}{2}$$

$$\mathrm{ch}(x+y)=\frac{e^{x+y}+e^{-(x+y)}}{2}=\frac{e^{x+y}+e^{-x-y}}{2}=\frac{1}{2}\left(e^{x+y}+e^{-x-y}\right)$$

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