DOMAČE NALOGE IZ PREDMETA
MATEMATIKA 2
E-VS

1. Dane so matrike $\mathbf{A} = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 0 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & 0 & 0 & 1 \\ -1 & 0 & 2 & -3 \\ 0 & -1 & 0 & -2 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$ in $\mathbf{C} = \begin{bmatrix} 4 & 3 \\ -4 & 2 \\ -1 & 0 \end{bmatrix}$.

   Izračunaj: $\mathbf{AB}$, $\mathbf{Ax}$, $\mathbf{x^Tx}$, $\mathbf{xx^T}$, $\mathbf{A} + \mathbf{C^T}$, $\mathbf{AB^T}$, $\mathbf{x^TC}$, $\mathbf{x^TA^TAx}$, $\mathbf{x^TA^TC^Tx}$, $\mathbf{A} + \mathbf{B}$, $\mathbf{x} + \mathbf{x^T}$ in $\mathbf{BA}$

   R : [$\mathbf{AB} = \begin{bmatrix} 3 & -3 & -2 & -2 \\ -4 & -1 & 0 & -4 \end{bmatrix}$, $\mathbf{Ax} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$, $\mathbf{x^Tx} = [14]$, $\mathbf{xx^T} = \begin{bmatrix} 9 & -3 & -6 \\ -3 & 1 & 2 \\ -6 & 2 & 4 \end{bmatrix}$, $\mathbf{A} + \mathbf{C^T} = \begin{bmatrix} 5 & -5 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $\mathbf{AB^T}$ ne obstaja, $\mathbf{x^TC} = [-18 \quad -7]$, $\mathbf{x^TA^TAx} = [68]$, $\mathbf{x^TA^TC^Tx} = [-92]$, $\mathbf{A} + \mathbf{B}$ ne obstaja, $\mathbf{x} + \mathbf{x^T}$ ne obstaja, $\mathbf{BA}$ ne obstaja].

2. Izračunaj $f(A)$, če je $f(x) = -2 - 5x + 3x^2$ za $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$.

   [$R : f(A) = \begin{bmatrix} 14 & 2 \\ 3 & 14 \end{bmatrix}$]

3. Reši naslednje sisteme linearnih enačb s pomočjo Gaussove metode:

   (a)

$$\begin{aligned} 2x - y + z &= 1 \\ x + 2y - z &= 2 \\ x - y + 2z &= 0 \end{aligned}$$

   [$R = \{(\frac{5}{6}, \frac{1}{2}, -\frac{1}{6})\}$]

   (b)

$$\begin{aligned} x + y - z &= 0 \\ 2x + 2y - 2z &= 0 \\ -x - y + z &= 1 \end{aligned}$$

   [R : sistem nima rešitve]

   (c)

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 + x_2 + x_3 &= 4 \\ x_2 + x_3 + x_4 &= -3 \\ x_3 + x_4 + x_5 &= 2 \\ x_4 + x_5 &= -1 \end{aligned}$$

   [$R = \{(6 - a, a - 5, 3, -1 - a, a);\ x_5 = a \in \mathbb{R}$ parameter $\}$]

4. Podana je matrika

$$\mathbf{A}(x) = \begin{bmatrix} 1 & 2 & -1 \\ 2x & 1 & -1 \\ 0 & x & 3 \end{bmatrix}$$

S pomočjo determinante ugotovi, za katero število $x \in \mathbb{R}$ matrika $\mathbf{A}(x)$ ni obrnljiva! Nato izračunaj $\mathbf{A}^{-1}(2)$.

[R : za $x_{1,2} = \frac{11 \pm \sqrt{145}}{-4}$ matrika ni obrnljiva, $\mathbf{A}^{-1}(2) = \frac{1}{-27}\begin{bmatrix} 5 & -8 & -1 \\ -12 & 3 & -3 \\ 8 & -2 & -7 \end{bmatrix}$]

5. Reši naslednje matrične enačbe:

(a) $\mathbf{A}^2\mathbf{X} = \mathbf{B}^2\mathbf{X} + \mathbf{A} - \mathbf{B}$, kjer je

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -3 & 3 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

[R : $\mathbf{X} = \frac{1}{75}\begin{bmatrix} -75 & 0 & -14 \\ 0 & -75 & 70 \\ 0 & 0 & 25 \end{bmatrix}$]

(b) $\mathbf{A}\mathbf{X} - 2\mathbf{X} = \mathbf{A} + \mathbf{I}$, kjer je

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$$

[R : $\mathbf{X} = \frac{1}{6}\begin{bmatrix} 4 & 6 \\ 8 & 12 \end{bmatrix}$]

6. Izračunaj naslednje determinante:

(a) $\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$ [R : 5]

(b) $\begin{vmatrix} 2 & 0 & 7 \\ 1 & 2 & 3 \\ 6 & 1 & -4 \end{vmatrix}$ [R : -99]

(c) $\begin{vmatrix} 1 & 0 & 3 & 4 \\ 1 & -1 & 2 & 5 \\ 7 & -2 & 3 & -4 \\ 0 & 1 & 6 & 8 \end{vmatrix}$ [R : 254]

MATEMATIKA 2 - DN E1054204 GREGOR NIKOLIĆ

1

$$\underset{2\,3}{A} = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 0 & 1 \end{bmatrix} \quad \underset{3\,4}{B} = \begin{bmatrix} 2 & 0 & 0 & 1 \\ -1 & 0 & 2 & -3 \\ 0 & -1 & 0 & -3 \end{bmatrix} \quad \underset{3\,1}{x} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} \quad \underset{3\,2}{C} = \begin{bmatrix} 4 & 3 \\ -4 & 2 \\ -1 & 0 \end{bmatrix}$$

$(1\cdot 2 + (-1)\cdot(-1) + 3\cdot 0), \dots$

$$\underset{2\,3\,3\,4}{AB} = \begin{bmatrix} 3 & -3 & -2 & -2 \\ -4 & -1 & 0 & -4 \end{bmatrix}$$

$(1\cdot(-3) + (-1)\cdot 1 + 1\cdot 2), \dots$

$$\underset{2\,3\,3\,1}{Ax} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$$

$(-3\cdot(-3) + 1\cdot 1 + 2\cdot 2)$

$$\underset{1\,3\,3\,1}{x^T x} = [14]$$

$$\underset{3\,1\,1\,3}{x x^T} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & -3 & -6 \\ -3 & 1 & 2 \\ -6 & 2 & 4 \end{bmatrix}$$

3×3

$$\underset{2\,3}{A} + \underset{2\,3}{C^T} = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -4 & -1 \\ 3 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\underset{2\,3\,4\,3}{AB^T} = /$$

ni enako, produkt ne obstaja

$$\underset{1\,3\,3\,2}{x^T C} = \begin{bmatrix} -3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ -4 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -18 & -7 \end{bmatrix}$$

1×2

se zmotil v napisu

$$\underset{1\,3\,3\,2}{x^T A^T} = \begin{bmatrix} -3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 8 \end{bmatrix}$$

$$\underset{1\,3\,3\,2\,2\,3\,3\,1}{x^T A^T A x} = \begin{bmatrix} -3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 3 \\ -2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$$

1×2 2×1

$$= \begin{bmatrix} 2 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 8 \end{bmatrix} = [68]$$

$$\underset{1\,3\,3\,2\,2\,3\,3\,1}{x^T A^T C^T x} = \begin{bmatrix} 2 & 8 \end{bmatrix} \cdot \begin{bmatrix} 4 & -4 & -1 \\ 3 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & 8 \end{bmatrix} \cdot \begin{bmatrix} -18 \\ -7 \end{bmatrix} = [-92]$$

1×2 2×1

$$\underset{2\,3\;3\,4}{A + B} = \text{ne obstaja} \quad \underset{3\,1\;1\,3}{x + x^T} = \text{ne obstaja} \quad \underset{3\,4\,2\,3}{BA} = \text{ne obstaja}$$

$\neq$ $\neq$ $\neq$

MATEMATIKA 2 - DN E1059204 GREGOR NIKOLIĆ

2

$f(A) = -2 - 5x + 3x^2 \qquad A = \begin{bmatrix}1&2\\3&1\end{bmatrix}$

Nikolić

$$-2I - 5A + 3AA = \begin{bmatrix}-2&0\\0&-2\end{bmatrix} - 5\begin{bmatrix}1&2\\3&1\end{bmatrix} + 3\begin{bmatrix}1&2\\3&1\end{bmatrix}\begin{bmatrix}1&2\\3&1\end{bmatrix} =$$

$$= -2I - \begin{bmatrix}5&10\\15&5\end{bmatrix} + 3\begin{bmatrix}7&4\\6&7\end{bmatrix} = -2I - \begin{bmatrix}5&10\\15&5\end{bmatrix} + \begin{bmatrix}21&12\\18&21\end{bmatrix} = \begin{bmatrix}-2&0\\0&-2\end{bmatrix} + \begin{bmatrix}16&2\\3&16\end{bmatrix} = \begin{bmatrix}14&2\\3&14\end{bmatrix}$$

3

a)

$$\left[\begin{array}{ccc|c}2&-1&1&1\\1&2&-1&2\\1&-1&2&0\end{array}\right] /\cdot(-1) \sim \left[\begin{array}{ccc|c}2&-1&1&1\\0&3&-3&2\\1&-1&2&0\end{array}\right] /\cdot(2)\ /\cdot(-1) \sim \left[\begin{array}{ccc|c}2&-1&1&1\\0&3&-3&2\\0&-\frac{1}{2}&\frac{3}{2}&-\frac{1}{2}\end{array}\right] /:6 \sim$$

$$\begin{array}{c}\begin{array}{ccc}x&y&z\end{array}\\ \sim \left[\begin{array}{ccc|c}2&-1&1&1\\0&3&-3&2\\0&0&1&-\frac{1}{6}\end{array}\right]\end{array}$$

$2x - y + z = 1 \Rightarrow x = \frac{5}{6}$

$\to 3y - 3z = 2 \Rightarrow y = \frac{1}{2}$

$\to z = -\frac{1}{6}$

$R: \left\{\left(\frac{5}{6}, \frac{1}{2}, -\frac{1}{6}\right)\right\}$

b)

$$\left[\begin{array}{ccc|c}1&1&-1&0\\2&2&-2&0\\-1&-1&1&1\end{array}\right] /\cdot 2 \sim \left[\begin{array}{ccc|c}1&1&-1&0\\0&0&0&2\\-1&-1&1&1\end{array}\right] \lightning$$

$0 \cdot x + 0 \cdot y + 0 \cdot z = 2$

Sistem nima rešitve!

3 MATEMATIKA 2 - DN E1054204 GREGOR NIKOLIĆ

c)
$$\begin{aligned}
x_1 + x_2 &= 1\\
x_1 + x_2 + x_3 &= 4\\
x_2 + x_3 + x_4 &= -3\\
x_3 + x_4 + x_5 &= 2\\
x_4 + x_5 &= -1
\end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1&1&0&0&0&1\\ 1&1&1&0&0&4\\ 0&1&1&1&0&-3\\ 0&0&1&1&1&2\\ 0&0&0&1&1&-1 \end{array}\right] /\cdot(-1) \sim \left[\begin{array}{ccccc|c} 1&1&0&0&0&1\\ 0&1&1&1&0&-3\\ 0&0&1&0&0&3\\ 0&0&1&1&1&2\\ 0&0&0&1&1&-1 \end{array}\right] /\cdot(-1) \sim \left[\begin{array}{ccccc|c} 1&1&0&0&0&1\\ 0&1&1&1&0&-3\\ 0&0&1&0&0&3\\ 0&0&0&1&1&-1\\ 0&0&0&1&1&-1 \end{array}\right] /\cdot(-1) \sim$$

$$\sim \begin{array}{c} \qquad\qquad\qquad\qquad a \\ \left[\begin{array}{ccccc|c} 1&1&0&0&0&1\\ 0&1&1&1&0&-3\\ 0&0&1&0&0&3\\ 0&0&0&1&1&-1\\ \cancel{0}&\cancel{0}&\cancel{0}&\cancel{0}&\cancel{0}&\cancel{0} \end{array}\right] \end{array} /\cdot(-1) \quad \begin{array}{c} \qquad\qquad\qquad\qquad a \text{ parametar} \\ \left[\begin{array}{ccccc|c} 1&1&0&0&0&1\\ 0&1&0&1&0&-6\\ 0&0&1&0&0&3\\ 0&0&0&1&1&-1\\ 0&0&0&0&0&0 \end{array}\right] \end{array}$$

odpade

$$\begin{aligned}
x_1 + x_2 &= 1 &\Rightarrow x_1 &= 1 - x_2 &\Rightarrow \boxed{x_1 = 6 - a}\\
x_2 + x_4 &= -6 &\Rightarrow x_2 &= -6 - x_4 &\Rightarrow \boxed{x_2 = a - 5}\\
x_3 &= 3 &\Rightarrow \boxed{x_3 = 3}\\
x_4 + x_5 &= -1 &\Rightarrow \boxed{x_4 = -1 - a}
\end{aligned}$$

$a \in \mathbb{R}$

parametar

$$R: \{(6-a,\ a-5,\ 3,\ -1-a,\ a)\}$$

4

MATEMATIKA 2 - DN E1054204 GREGOR NIKOLIĆ

$$A(x) = \begin{bmatrix} 1 & 2 & -1 \\ 2x & 1 & -1 \\ 0 & x & 3 \end{bmatrix}$$

$$\det A(x) = 1\cdot\left(1\cdot 3-(x\cdot(-1))\right) - 2x\left(2\cdot 3-(x\cdot(-1))\right) = -2x^2-11x+3$$

$$x_{1,2} = \frac{11\pm\sqrt{145}}{-4}$$

$$A^{-1}_{ij} = \frac{1}{\det A}\cdot \tilde{A}^T$$

$$A(2) = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 1 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$

$$A_{ij} = (-1)^{i+j}\cdot \overbrace{\det A_{ij}}^{\text{poddeterminanta } a_{ij}}$$

$$A_{11} = (-1)^2\cdot\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = (1\cdot 3-(-1\cdot 2)) = \boxed{5}$$

$$A_{12} = (-1)^3\cdot\begin{bmatrix} 4 & -1 \\ 0 & 3 \end{bmatrix} = -1(3\cdot 4) = \boxed{-12}$$

$$A_{13} = (-1)^4\cdot\begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix} = 2\cdot 4 = \boxed{8}$$

$$A_{21} = (-1)^3\cdot\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix} = -1(6-(-2)) = \boxed{-8}$$

$$A_{22} = (-1)^4\cdot\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} = \boxed{3}$$

$$A_{23} = (-1)^5\cdot\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} = -1(2) = \boxed{-2}$$

$$A_{31} = (-1)^4\cdot\begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix} = -2-(-1) = \boxed{-1}$$

$$A_{32} = (-1)^5\cdot\begin{bmatrix} 1 & -1 \\ 4 & -1 \end{bmatrix} = -1(-1-(-4)) = \boxed{-3}$$

$$A_{33} = (-1)^6\cdot\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = 1-8 = \boxed{-7}$$

$$\det A(2) = 1\cdot 3-(-2)-4(6-(-2)) = 5-32 = -27$$

$$A = \begin{bmatrix} 5 & -12 & 8 \\ -8 & 3 & -2 \\ -1 & -3 & -7 \end{bmatrix} \quad A^T = \begin{bmatrix} 5 & -8 & -1 \\ -12 & 3 & -3 \\ 8 & -2 & -7 \end{bmatrix}$$

$$A^{-1}(2) = \frac{1}{-27}\begin{bmatrix} 5 & -8 & -1 \\ -12 & 3 & -3 \\ 8 & -2 & -7 \end{bmatrix}$$

MATEMATIKA 2 - DN   E1054204

GREGOR NIKOLIĆ

5

a) $A = \begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} -3 & 3 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

$$A^2X = B^2X + A - B \Rightarrow (A^2 - B^2)X = A - B \quad (A^2-B^2)^{-1} \cdot \backslash$$

$$\underbrace{IX}_{X} = (A^2-B^2)^{-1} \cdot (A-B)$$

$$A - B = \begin{bmatrix} 5 & 1 & -2 \\ 0 & -5 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -4 & 5 \\ 0 & 9 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^2 = BB = \begin{bmatrix} -3 & 3 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -3 & 3 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 9 & -3 & 11 \\ 0 & 4 & 16 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} -5 & -1 & -6 \\ 0 & 5 & -20 \\ 0 & 0 & -3 \end{bmatrix}$$

$$(A^2-B^2)^{-1} = \left[\begin{array}{ccc|ccc} -5 & -1 & -6 & 1 & 0 & 0 \\ 0 & 5 & -20 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} /:(-5) \\ /:(5) \\ /:(-3) \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & \frac{1}{5} & \frac{6}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 1 & -4 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} \end{array}\right] /\cdot 4 \; + \sim \left[\begin{array}{ccc|ccc} 1 & \frac{1}{5} & \frac{6}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{5} & -\frac{4}{3} \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} \end{array}\right] \begin{array}{l} \\ /\cdot(-\frac{1}{5}) \\ /\cdot(-\frac{6}{5}) \end{array} + \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{5} & -\frac{1}{25} & \frac{10}{15} \\ 0 & 1 & 0 & 0 & \frac{1}{5} & -\frac{4}{3} \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} \end{array}\right]$$

$-\frac{1}{5} \cdot \frac{1}{5}$ $\quad -\frac{2}{5} + \frac{4}{3} = \frac{-6+20}{15} = \frac{14}{15}$

$\frac{2}{5} + \frac{2}{25} - \frac{10}{15} = \frac{30+6-50}{75} = -\frac{14}{75}$

$$X = (A^2-B^2)^{-1} \cdot (A-B) = \begin{bmatrix} -\frac{1}{5} & -\frac{1}{25} & \frac{10}{15} \\ 0 & \frac{1}{5} & -\frac{4}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 & -2 \\ 0 & -5 & -2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -\frac{14}{75} \\ 0 & -1 & \frac{14}{15} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} /\cdot 5$$

$$= \frac{1}{75} \begin{bmatrix} -75 & 0 & -14 \\ 0 & -75 & 70 \\ 0 & 0 & 25 \end{bmatrix}$$

5 MATEMATIKA 2 - DN E1054204 GREGOR NIKOLIĆ

b) $AX - 2X = A + I \qquad A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

$(A-2I)X = A + I \quad (A-2I)^{-1} \cdot \backslash$

$\underbrace{IX}_{X} = (A-2I)^{-1}(A+I)$

$X = (A-2I)^{-1} \cdot (A+I)$

$A + I = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

$A - 2I = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

$(A-2I)^{-1} = \left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{array}\right] /\cdot(2) + \sim \left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 0 & 6 & 2 & 1 \end{array}\right] \begin{array}{l} /\cdot(-1) \\ /:6 /\cdot(3) \end{array} + \sim$

$\sim \left[\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{1}{6} \end{array}\right] \Rightarrow (A-2I)^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$

$X = (A-2I)^{-1} \cdot (A+I)$

$X = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6 & 9 \\ 6 & 9 \end{bmatrix} = \begin{bmatrix} 1 & \frac{3}{2} \\ 1 & \frac{3}{2} \end{bmatrix}$

⑥ MATEMATIKA 2 - DN E1054204 GREGOR NIKOLIĆ

a) $\begin{vmatrix}2&3\\1&4\end{vmatrix} = 2\cdot 4 - 3 = \underline{\underline{5}}$

b)
$$\begin{array}{c|ccc|}
 & + & - & + \\
+ & 2 & 0 & 7 \\
- & 1 & 2 & 3 \\
+ & 6 & 1 & -4
\end{array} = 2\begin{bmatrix}2&3\\1&-4\end{bmatrix} + 7\begin{bmatrix}1&2\\6&1\end{bmatrix} = 2(2\cdot(-4)-3) + 7(1-12) = -22-77 = \underline{\underline{-99}}$$

c)
$$\begin{array}{c|cccc|}
 & + & - & + & - \\
+ & 1 & 0 & 3 & 4 \\
- & 1 & -1 & 2 & 5 \\
+ & 7 & -2 & 3 & -4 \\
- & 0 & 1 & 6 & 8
\end{array} = -\begin{bmatrix}-1&2&5\\-2&3&-4\\1&6&8\end{bmatrix} + 3\begin{bmatrix}1&-1&5\\7&-2&-4\\0&1&8\end{bmatrix} - 4\begin{bmatrix}1&-1&2\\7&-2&3\\0&1&6\end{bmatrix} =$$

$$= \underbrace{-1\big(3\cdot 8-(-4\cdot 6)\big)}_{-48}\ \underbrace{-2\big(-2\cdot 8-(-4\cdot 1)\big)}_{24}\ \underbrace{+5\big(-2\cdot 6-(3\cdot 1)\big)}_{-75}\ \underbrace{+3\Big(\big(-2\cdot 8-(-4\cdot 1)\big)-7\cdot(-8-5)\Big)}_{237} -$$

$$\underbrace{-4\cdot\Big(\big(-2\cdot 6-(3)\big)-7\cdot(-6-2)\Big)}_{-164} =$$

$$= -48+24-75+237-164 = \underline{\underline{-26}}$$