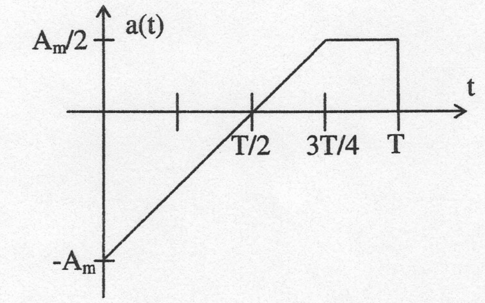
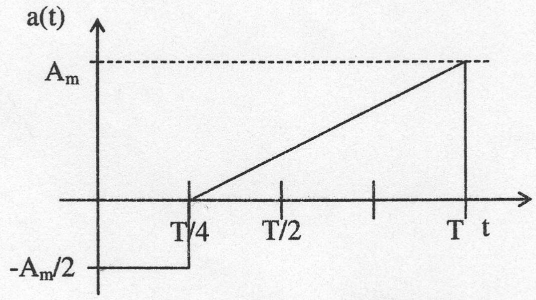
1 Domača naloga iz Osnov elektrotehnike II

1. Izračunajte (po definiciji) aritmetično in efektivno srednjo vrednost signala na sliki 1.

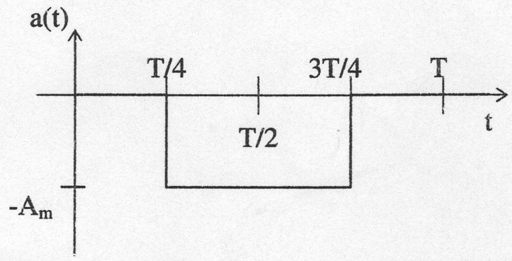


2. Izračunajte (po definiciji) aritmetično in efektivno srednjo vrednost signala na sliki 1.

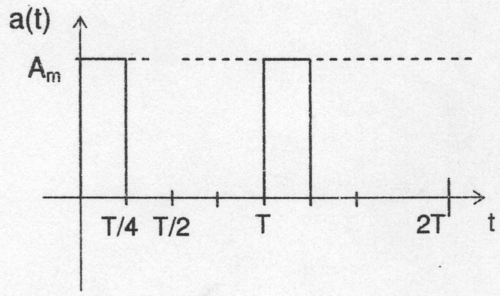


Slika 1

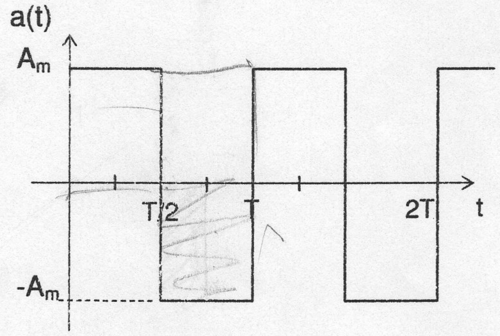
3. Razvijte signal na sliki v Fourierjevo vrsto.



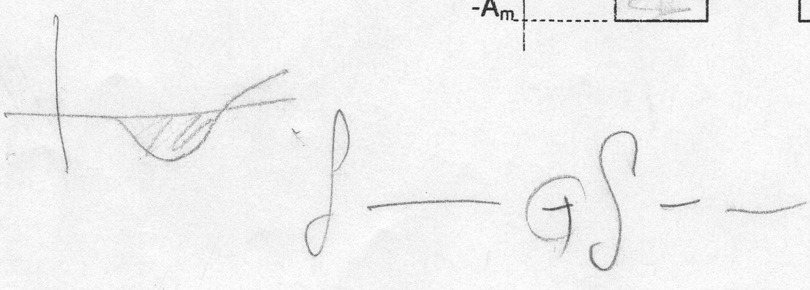
4. Razvijte signal na sliki v Fourierjevo vrsto.



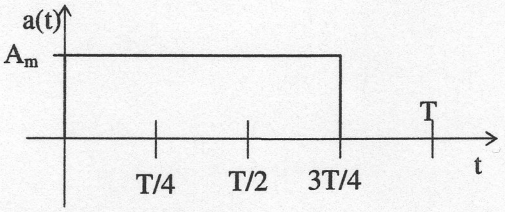
5. Razvijte signal na sliki v Fourierjevo vrsto.



"tudi" bit rešene naloge!



6. Razvijte signal na sliki v Fourierjevo vrsto.



Rešitve:

1. $A_{sr} = -\frac{A_m}{16} \qquad A = \frac{A_m}{2}$

2. $A_{sr} = \frac{A_m}{4} \qquad A = \frac{A_m\sqrt{5}}{4}$

3.
$$A_0 = -A_m$$
$$B_n = \frac{-Am}{n\cdot\pi}\left(\sin\left(n\,\frac{3\pi}{2}\right) - \sin\left(n\,\frac{\pi}{2}\right)\right)$$
$$a(t) = -\frac{Am}{2} + \frac{2A_m}{\pi}\left(\cos(\omega t) - \tfrac{1}{3}\cos(3\omega t) + \tfrac{1}{5}\cos(5\omega t) - \ldots\right)$$

4.
$$A_0 = \frac{A_m}{2}$$
$$A_n = \frac{A_m}{n\cdot\pi}\left(1 - \cos\left(n\frac{\pi}{2}\right)\right)$$
$$B_n = \frac{Am}{n\cdot\pi}\,\sin\left(n\,\frac{\pi}{2}\right)$$
$$a(t) = \frac{Am}{4} + \frac{A_m}{\pi}\left(\sin(\omega t) + \sin(2\omega t) + \tfrac{1}{3}\sin(3\omega t) + \tfrac{1}{5}\sin(5\omega t) + \ldots\right) +$$
$$+ \frac{Am}{\pi}\left(\cos(\omega t) - \tfrac{1}{3}\cos(3\omega t) + \tfrac{1}{5}\cos(5\omega t)\right)$$

5.
$$A_n = \frac{2A_m}{n\cdot\pi}\left(1 - \cos(n\pi)\right)$$
$$a(t) = \frac{4A_m}{\pi}\left(\sin(\omega t) + \tfrac{1}{3}\sin(3\omega t) + \tfrac{1}{5}\sin(5\omega t) + \ldots\right)$$

$A_0 +$
A_m

6.
$$A_n = \frac{A_m}{n\;\pi}\left(1 - \cos(n\frac{3\pi}{2})\right)$$
$$B_n = \frac{A_m}{n\cdot\pi}\left(\sin(n\frac{3\pi}{2})\right)$$
$$a(t) = \frac{3}{4}A_m + \frac{A_m}{\pi}\left(\sin(\omega t) + \sin(2\omega t) + \tfrac{1}{3}\sin(3\omega t) + \tfrac{1}{5}\sin(5\omega t) + \ldots\right) +$$
$$+ \frac{A_m}{\pi}\left(-\cos(\omega t) + \tfrac{1}{3}\cos(3\omega t) - \tfrac{1}{5}\cos(5\omega t) + \ldots\right)$$

DN. 1/1 OEZG

$$a(t)=\begin{cases}\frac{2A_m}{T}\cdot t-A_m, & 0\le t\le \frac{3T}{4}\\ \frac{A_m}{2}; & \frac{3T}{4}\le t\le T\end{cases}$$

$$A_{sr}=\frac{1}{T}\int_0^T a(t)\,dt$$

$$A_{sr}=\frac{1}{T}\left(\int_0^{\frac{3T}{4}}\left(\frac{2A_m}{T}t-A_m\right)dt+\int_{\frac{3T}{4}}^{T}\frac{A_m}{2}\,dt\right)$$

$$=\frac{1}{T}\left(\left(\frac{2A_m}{T}\frac{t^2}{2}-A_m\cdot t\right)\Bigg|_0^{\frac{3T}{4}}+\frac{A_m}{2}\cdot t\Bigg|_{\frac{3T}{4}}^{T}\right)$$

$$=\frac{1}{\cancel{T}}\left(\frac{\cancel{2}A_m}{\cancel{2}\cancel{T}}\cdot\frac{9\cancel{T^2}}{16}-A_m\cdot\frac{3\cancel{T}}{4}+\frac{A_m}{2}\cdot\frac{\cancel{T}}{4}\right)$$

$$\frac{9A_m}{16}-\frac{3A_m}{4}+\frac{A_m}{8}$$

$$=\frac{A_m}{16}\left(9-12+2\right)$$

$$=-\frac{A_m}{16}\ V$$

$$A^2=\frac{1}{T}\int_0^T a^2(t)\,dt$$

$$A^2=\frac{1}{T}\left(\int_0^{\frac{3T}{4}}\left(\frac{2A_m}{T}t-A_m\right)^2dt+\int_{\frac{3T}{4}}^{T}\left(\frac{A_m}{2}\right)^2dt\right)$$

$$=\frac{1}{T}\left(\int_0^{\frac{3T}{4}}\left(\frac{4A_m^2}{T^2}t^2-\frac{4A_m^2t}{T}+A_m^2\right)dt+\int_{\frac{3T}{4}}^{T}\frac{A_m^2}{4}\,dt\right)$$

$$=\frac{1}{T}\left(\frac{4A_m^2}{T^2}\frac{t^3}{3}-\frac{4A_m^2}{T}\frac{t^2}{2}+A_m^2t\Bigg|_0^{\frac{3T}{4}}+\frac{A_m^2}{4}t\Bigg|_{\frac{3T}{4}}^{T}\right)\qquad \frac{4T}{4}-\frac{3T}{4}=\frac{T}{4}$$

$$A^2=\frac{1}{\cancel{T}}\left(\frac{4A_m^2}{\cancel{3}\cancel{T^2}}\cdot\frac{\cancel{27}^{9}\cancel{T^3}}{\cancel{64}\,16}-\frac{\cancel{4}A_m^2}{\cancel{2}\cancel{T}}\cdot\frac{9\cancel{T^2}}{\cancel{16}\,8}+\frac{3A_m^2\cancel{T}}{4}+\frac{A_m^2\cancel{T}}{16}\right)$$

$$A^2=\frac{9A_m^2}{16}-\frac{9A_m^2}{8}+\frac{3A_m^2}{4}+\frac{A_m^2}{16}$$

$$A^2=\frac{A_m^2}{16}\left(9-18+12+1\right)=\frac{4A_m^2}{16}\Rightarrow A=\frac{2A_m}{4}=\frac{A_m}{2}\ V$$

OELE DN 1 /2

$$a(t)=\begin{cases}-\dfrac{A_m}{2}, & 0\le t\le \dfrac{T}{4}\\[2mm] \dfrac{4A_m}{3T}\cdot t-\dfrac{A_m}{3}, & \dfrac{T}{4}\le t< T\end{cases}$$

$$A_{sr}=\frac{1}{T}\int_0^T a(t)\,dt$$

$$A_{sr}=\frac{1}{T}\left(\int_0^{\frac{T}{4}}-\frac{A_m}{2}\,dt+\int_{\frac{T}{4}}^{T}\left(\frac{4A_m}{3T}\cdot t-\frac{A_m}{3}\right)dt\right)=\frac{1}{T}\left(-\frac{A_m}{2}\cdot t\Big|_0^{\frac{T}{4}}+\frac{4A_m}{3T}\cdot\frac{t^2}{2}-\frac{A_m}{3}\cdot t\Big|_{\frac{T}{4}}^{T}\right)$$

$$=\frac{\cancel{1}}{\cancel{T}}\left(-\frac{A_m}{2}\,\frac{\cancel{T}}{4}+\frac{\cancel{4}A_m}{2\cancel{6}\cancel{T}}\,\frac{\overset{5}{\cancel{15}}\cancel{T^2}}{\cancel{16}4}-\frac{A_m}{\cancel{3}}\cdot\frac{\cancel{3}\cancel{T}}{4}\right)=\frac{A_m}{8}\Big(\underbrace{1+5-2}_{2}\Big)=\frac{\cancel{2}A_m}{\cancel{8}4}\Rightarrow \underline{\underline{A_{sr}=\frac{A_m}{4}}}$$

$$A^2=\frac{1}{T}\int_0^T a^2(t)\,dt$$

$$A^2=\frac{1}{T}\left(\int_0^{\frac{T}{4}}\left(-\frac{A_m}{2}\right)^2dt+\int_{\frac{T}{4}}^{T}\left(\frac{4A_m}{3T}\cdot t-\frac{A_m}{3}\right)dt\right)$$

$$\frac{1}{T}\left(\int_0^{\frac{T}{4}}\frac{A_m^2}{4}\,dt+\int_{\frac{T}{4}}^{T}\left(\frac{16A_m^2}{9T^2}\cdot t^2-\frac{8A_m^2}{9T}\cdot t+\frac{A_m^2}{9}\right)dt\right)$$

$$=\frac{1}{T}\left(\frac{A_m^2}{4}\cdot t\Big|_0^{\frac{T}{4}}+\left(\frac{16A_m^2}{9T^2}\cdot\frac{t^3}{3}-\frac{8A_m^2}{9T}\cdot\frac{t^2}{2}+\frac{A_m^2}{9}\cdot t\right)\Big|_{\frac{T}{4}}^{T}\right)$$

$$=\frac{1}{T}\left(\frac{A_m^2}{4}\cdot\frac{T}{4}+\frac{16A_m^2}{27T^2}\cdot\underbrace{\left(T^3-\frac{T^3}{64}\right)}_{\frac{63T^3}{64}}-\frac{8A_m^2}{18T}\underbrace{\left(T^2-\frac{T^2}{16}\right)}_{\frac{15T^2}{16}}+\frac{A_m^2}{9}\,\frac{3T}{4}\right)$$

$$=\frac{\cancel{1}}{\cancel{T}}\left(\frac{A_m^2\cancel{T}}{16}+\frac{\overset{1}{\cancel{16}}A_m^2}{9\,\cancel{27}\cancel{T^2}}\cdot\frac{\overset{21}{\cancel{63}}\cancel{T^3}}{\underset{4}{\cancel{64}}}-\frac{\overset{1}{\cancel{8}}A_m^2}{\underset{6}{\cancel{18}}\cancel{T}}\cdot\frac{\overset{5}{\cancel{15}}\cancel{T^2}}{\underset{2}{\cancel{16}}}+\frac{A_m^2}{\underset{3}{\cancel{9}}}\,\frac{\overset{1}{\cancel{3}}\cancel{T}}{4}\right)$$

$$=\frac{A_m^2}{16}+\frac{\overset{7}{\cancel{21}}A_m^2}{\cancel{36}_{12}}-\frac{5A_m^2}{12}+\frac{A_m^2}{12}=\frac{A_m^2}{48}\left(3+\cancel{2}8-\cancel{2}0+4\right)=\frac{A_m^2\,\cancel{15}\,5}{\cancel{48}\,16}$$

$$\Rightarrow \underline{\underline{A=\frac{A_m\sqrt{5}}{4}}}$$

OE1LE DN 1/3

$$a(t) = -A_m \quad ; \quad \frac{T}{4} \le t \le \frac{3T}{4}$$

$$A_0 = \frac{2}{T}\int_{\frac{T}{4}}^{\frac{3T}{4}} -A_m \, dt = \frac{2}{\cancel{T}} \cdot -A_m \cdot \frac{\cancel{T}}{2} = \boxed{-A_m}$$

• Funkcija je soda (preko ordinatne osi) zato odpadejo členi A_n:

Dokaz:

$$\frac{2\pi}{\cancel{T}} \cdot \frac{3\cancel{T}}{12} = \frac{3\pi}{2} \qquad \frac{2\pi}{\cancel{T}} \cdot \frac{\cancel{T}}{12} = \frac{\pi}{2}$$

$$A_n = \frac{2}{T}\int_{\frac{T}{4}}^{\frac{3T}{4}} -A_m \cdot \sin(n\omega t) = \frac{+2A_m}{Tn\omega} \cdot \cos(n\omega t)\Big|_{\frac{T}{4}}^{\frac{3T}{4}} = \frac{A_m}{n\pi} \cdot \underbrace{\left(\cos\left(n\frac{3\pi}{2}\right) - \cos\left(n\frac{\pi}{2}\right)\right)}_{0}$$

$$B_n = \frac{2}{T}\int_{\frac{T}{4}}^{\frac{3T}{4}} -A_m \cos(n\omega t) = \frac{-2A_m}{Tn\omega} \sin(n\omega t)\Big|_{\frac{T}{4}}^{\frac{3T}{4}} = -\frac{A_m}{n\pi} \cdot \underbrace{\left(\sin\left(n\frac{3\pi}{2}\right) - \sin\left(n\frac{\pi}{2}\right)\right)}_{-2,0,2,0,\ldots}$$

$$\boxed{B_n = \frac{-A_m}{n\pi}\left(\sin\left(n\frac{3\pi}{2}\right) - \sin\left(n\frac{\pi}{2}\right)\right)}$$

$$B_1 = \frac{2A_m}{\pi} \quad B_2 = 0 \quad B_3 = \frac{-2A_m}{3\pi} \quad B_4 = 0 \quad B_5 = \frac{2A_m}{5\pi} \quad B_6 = 0$$

$$\boxed{a(t) = \frac{A_m}{2} + \frac{2A_m}{\pi}\left(\cos(\omega t) - \frac{1}{3}\cos(3\omega t) + \frac{1}{5}\cos(5\omega t) - \ldots\right)}$$

OELE DN①/4

$$a(t) = A_m \,, \quad 0 \le t \le \frac{T}{4}$$

$$A_0 = \frac{2}{T}\int_0^{\frac{T}{4}} A_m \, dt = \frac{2A_m}{\cancel{T}} \cdot \frac{\cancel{T}}{4} = \boxed{\frac{A_m}{2}}$$

$$\frac{2\pi}{T} \cdot \frac{T}{4} = \frac{\pi}{2}$$

$$A_n = \frac{2}{T}\int_0^{\frac{T}{4}} A_m \sin(n\omega t)\, dt = \frac{-A_m}{n\pi} \cdot \cos(n\omega t)\Big|_0^{\frac{T}{4}} = \frac{-A_m}{n\pi} \cdot \left(\cos\left(n\frac{\pi}{2}\right) - 1\right)$$

$$\boxed{A_n = \frac{A_m}{n\pi} \cdot \left(1 - \cos\left(n\frac{\pi}{2}\right)\right)}$$

$$B_n = \frac{2}{T}\int_0^{\frac{T}{4}} A_m \cos(n\omega t)\, dt = \frac{A_m}{n\pi} \cdot \sin(n\omega t)\Big|_0^{\frac{T}{4}} = \frac{A_m}{n\pi} \cdot \sin\left(n\frac{\pi}{2}\right)$$

$$\boxed{B_n = \frac{A_m}{n\pi} \sin\left(n\frac{\pi}{2}\right)}$$

$A_1 = \frac{A_m}{\pi}$ $\qquad$ $B_1 = \frac{A_m}{\pi}$

$A_2 = \frac{A_m}{\pi}$ $\qquad$ $B_2 = \emptyset$

$A_3 = \frac{A_m}{3\pi}$ $\qquad$ $B_3 = \frac{A_m}{3\pi}$

$A_4 = \emptyset$ $\qquad$ $B_4 = \emptyset$

$A_5 = \frac{A_m}{5\pi}$ $\qquad$ $B_5 = \frac{A_m}{5\pi}$

$$\boxed{\begin{aligned} a(t) = \frac{A_m}{4} &+ \frac{A_m}{\pi}\left(\sin(\omega t) + \sin(2\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) \ldots\right) + \\ &\frac{A_m}{\pi}\left(\cos(\omega t) - \frac{1}{3}\cos(3\omega t) + \frac{1}{5}\cos(5\omega t) - \ldots\right) \end{aligned}}$$

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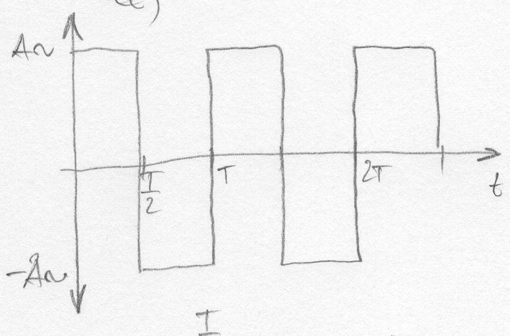
$$a(t) = \begin{cases} +A_m \; ; \; 0 \le t \le \frac{T}{2} \\ -A_m \; ; \; \frac{T}{2} \le t < T \end{cases}$$



~ Liha funkcija → odpadejo B členi!

$$A_0 = \frac{2}{T}\left(\int_0^{\frac{T}{2}} A_m\,dt - \int_{\frac{T}{2}}^{T} -A_m\,dt\right) = \frac{2}{T}\left(\frac{A_m T}{2} + \frac{A_m T}{2}\right) = \frac{2A_m \cancel{T}}{\cancel{T}} = \boxed{2A_m}$$

$$A_n = \frac{2}{T}\left(\int_0^{\frac{T}{2}} A_m \sin(n\omega t)\,dt - \int_{\frac{T}{2}}^{T} \left(-A_m \sin(n\omega t)\right)dt\right)$$

$$A_n = \frac{2}{T}\left(-\frac{A_m}{n\omega} \cos(n\omega t)\Big|_0^{\frac{T}{2}} \quad \left(\frac{A_m}{n\omega} \cos(n\omega t)\right)\Big|_{\frac{T}{2}}^{T}\right)$$

$$A_n = \frac{2}{T}\left(\frac{A_m}{n\omega}\left(\cos(n\pi) \quad \underbrace{\cos(0)}_{1}\right) - \left(\frac{A_m}{n\omega}\left(\underbrace{\cos(n2\pi)}_{1} \quad \cos(n\pi)\right)\right)\right)$$

$$A_n = \frac{2}{T}\left(-\frac{A_m}{n\omega}\left(\cos(n\pi) \; 1 - 1 + \cos(n\pi)\right)\right)$$

$$A_n = \frac{\cancel{2}A_m}{\cancel{2}\pi n}\left(2\cos(n\pi) \quad 2\right) = \boxed{\frac{2A_m}{n\pi}\left(1 \quad \cos(n\pi)\right)}$$

$$\boxed{A_1 = \frac{2A_m}{\pi} \cdot 2 = \frac{4A_m}{\pi}}, \quad A_2 = \frac{A_m}{\pi} \cdot \emptyset = \emptyset \quad \boxed{A_3 = \frac{2A_m}{3\pi} \; 2 = \frac{4A_m}{3\pi}}$$

$$A_4 = \frac{A_m}{2\pi} \; \emptyset = \emptyset \quad , \quad \boxed{A_5 = \frac{2A_m}{5\pi} \cdot 2 = \frac{4A_m}{5\pi}}$$

$$a(t) = A_m + \frac{4A_m}{\pi} \sin(\omega t) + \frac{4A_m}{3\pi} \sin(3\omega t) + \frac{4A_m}{5\pi} \sin(5\omega t)$$

$$a(t) = A_m + \frac{4A_m}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \ldots\right)$$

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$a(t) = A_m \qquad 0 \le t \le \frac{3T}{4}$

$\frac{2\pi}{\cancel{T}} \cdot \frac{3\cancel{T}}{\cancel{4}2} = \frac{3\pi}{2}$

$$A_0 = \frac{2}{T}\int_0^{\frac{3T}{4}} A_m \, dt = \frac{\cancel{2}A_m}{\cancel{T}} \cdot \frac{3\cancel{T}}{\cancel{4}2} = \frac{3A_m}{2}$$

$$A_n = \frac{2}{T}\int_0^{\frac{3T}{4}} A_m \sin(n\omega t) = \frac{-A_m}{n\pi} \cdot \cos(n\omega t)\Big|_0^{\frac{3T}{4}} = \frac{-A_m}{n\pi} \cdot \left(\cos\left(n\frac{3\pi}{2}\right) - \cos(0)\right)$$

$$\boxed{A_n = \frac{A_m}{n\pi}\left(1 \quad \cos\left(\frac{n\pi}{2}\right)\right)}$$

$$B_n = \frac{2}{T}\int_0^{\frac{3T}{4}} A_m \cos(n\omega t) = \frac{A_m}{n\pi} \cdot \sin(n\omega t)\Big|_0^{\frac{3T}{4}} = \frac{A_m}{n\pi} \cdot \sin\left(n\frac{3\pi}{2}\right)$$

$$\boxed{B_n = \frac{A_m}{n\pi} \sin\left(\frac{n3\pi}{2}\right)}$$

$A_1 = \frac{A_m}{\pi} \qquad B_1 = \frac{A_m}{\pi}$

$A_2 = \frac{\cancel{2}A_m}{2\pi} \qquad B_2 = \emptyset$

$A_3 = \frac{A_m}{3\pi} \qquad B_3 = \frac{A_m}{3\pi}$

$A_4 = \emptyset \qquad B_4 = \emptyset$

$A_5 = \frac{A_m}{5\pi} \qquad B_5 = \frac{-A_m}{5\pi}$

$$a(t) = \frac{3A_m}{4} + \frac{A_m}{\pi}\left(\sin(\omega t) + \sin(2\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \dots\right) +$$

$$\frac{A_m}{\pi}\left(\cos(\omega t) + \frac{1}{3}\cos(3\omega t) - \frac{1}{5}\cos(5\omega t) + \dots\right)$$